



## MATHEMATICAL MODELING AND SIMULATION OF A 3 DOFS LOWER LIMB REHABILITATION ROBOT FOR POST-STROKE PATIENT

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### Abstract:

*Stroke is a sickness that has a high potential of causing disability in the aged. Post-stroke patients can improve their motional functions by the rehabilitation program. With the development of science and technology, rehabilitation exercises can be performed by robots. This paper presents the process of modeling the mathematical model of a 3 degree of freedom (DOF) lower limb rehabilitation, including: kinematic and dynamic model, friction model and motor model. The simulation in MATLAB/Simulink is also carried out to show accuracy and suitability with the physical characteristics of the system model. This result is used to propose control strategies for robot in next studies.*

**Keywords:** motor function, rehabilitation robot, kinematic model, dynamic model, simulation.

### 1. Introduction

Stroke is a brain attack. It can happen to anyone at any time. Nowadays, stroke becomes a common sickness and it is one of the major cause of motor dysfunction, even permanent disability. Some people recover completely from stroke, but more than 2/3 of survivors will have some type of disability [1]. Among the motor disabilities of the post-stroke patient, lower limb disability is the second most affected part with 72% of the post-stroke patient and becomes the biggest obstacle in the daily life which seriously affecting quality of their life [2]. Thus, improving lower limb rehabilitation for post-stroke patients becomes an important and urgent issue that must be handled [3].

Study [4] has shown that post-stroke patients can improve their lower limb motional functions after they participate in rehabilitation programs. Treatments with specific, repetitive, high-intensity exercises are an effective way to improve lower limb motions after stroke [5]. The goal of a rehabilitation program is to help post-stroke patient relearn lost skills when stoke affected part of their lower limb. The focus of the rehabilitation program is to maintain joints in the patient's lower limb by passive motions, which includes continuous passive motions. Rehabilitation programs can be performed by physical therapy which is given by trained physiotherapists and medical professionals. The number of physiotherapists is severely lacking, the evaluation methods are mostly subjective and

do not ensure the accurate imitation of the required pattern of the exercise due to human errors, so the treatment effects cannot be guaranteed. Moreover, lower limb rehabilitation is a long-term process that is often divided into several stages, the application of high-intensity passive repetitive motion designed for patients in specific stages are necessary. Thus, there arises the requirement in developing assistive device, which are capable of providing better therapeutics [6].

At recent decades, the robotic rehabilitation field has evolved and many rehabilitation robots have been developed by companies or research groups, for example: Lokomat, Lokohelp, Alex, Lopes... Lokomat is a typical treadmill base exoskeleton robots with body weight support mechanic. It provides powered assistance at the hip and knee by strapping patient's legs. Hip and knee joint angles can also be adjusted individually during training to suit the specific needs of the patient. Patient co-operation method is used in Lokomat, and it has been demonstrated through good recovery signs [7]. Lokohelp is a 2 DOFs lower limb rehabilitation robot with structure similar to Lokomat. It can be placed on a treadmill with weight support mechanism and transmits movement of treadmill to levers for patients to track [8]. Alex [9] is a powered leg orthosis with actuators at hip and knee joint. It has a gait training method, can help post-stroke patients to become more healthy. Lopes is also a typical treadmill base exoskeleton robots

which has three actuated rotational joints: two at the hip and one at the knee. It can move in parallel with the legs when walking on a treadmill [10]. In [11] proposed an exoskeleton of 4 DOFs for hip, knee and ankle joints in the planar plane using force and impedance parameters for both robot and human. However, the authors only tested it in knee and ankle joints. Chao Zhang and colleagues introduced an exoskeleton of hip and knee joint rehabilitation functions based on detecting and tracking the movement of healthy legs used as control signals for disable legs, using elastic actuators to detect the torque in the healthy leg accurately used to calculate the support torque on the paralyzed with parameters corresponding to the patient's recovery level. The level of recovery and condition of patients is monitored and evaluated in real time [12]. In the joint training, NEXOS and TEM are typical joint rehabilitation systems. NEXOS is a system that can operate independently or provide direct support to a physiotherapist. It is a low-cost design, used in at the patient's using web-based strategies [13]. TEM is a device which aims to help stroke patients to recover their motor functions using kinetic therapy in the hip joint and knee joint by moving the lower limb using two mechanical arms [14]. An active rehabilitation joint device in [15] is a parallel 3-RPR functional rehabilitation robot with 1 DOF for hip joints and 2 DOFs for knee joints in the plane. The length of the outer skeleton can be adjusted steplessly to match the height of each patients. A 3 DOFs lower limb rehabilitation robot (LLRR) was designed and developed by Jinwu Gao's team [16] to help post-stroke patients in rehabilitation of the hip, knee and ankle joints. The size of its links can be also adjusted to match the height of each patient. Recently, a passive serial orthosis for a foot-plate based sitting-type lower limb rehabilitation robot is proposed by JK. Mohanta et al. It is the hybrid manipulator having 3 DOFs which records the hip, knee and ankle joint movement [17].

The robots mentioned above are all designed with a treadmill or at least a necessary physical therapy method when training. These robots are complex, cost-effective and offer good performance in appearance recovery, but are not suitable for single-joint training. Robot like TEM is not designed for the three main joints of the lower limb. These robots cannot train full-body limb rehabilitation. Moreover, some robots are not suitable for different patients because the patient's changeable states and devices are rarely considered or mentioned in studies.

This paper proposed a mechanical structure

of 3 DOFs robot for rehabilitation. We also presents the process of modeling and building simulation model of a 3 DOFs rehabilitation robot in MATLAB/Simulink. Motion and force control algorithms will be presented in next studies.

## 2. Mathematical Model

### 2.1. Mechanical structure and linkage diagram

The movement of human's lower limbs without movements of toes consists of 4 degrees of freedom in the ankles, knee and hip joints, in which the hip joint has 2 degrees of freedom. However, through survey of rehabilitation programs, the mainly practiced in the hip joint is usually flexion, so we determined the mechanical structure of robot is a 3 DOFs structure. Mechanical structure of robot is given in Fig. 1. It is comprised of three parts: Thigh, leg and foot, in which length of thigh and leg will be designed to be adjustable based on the heights of patients.

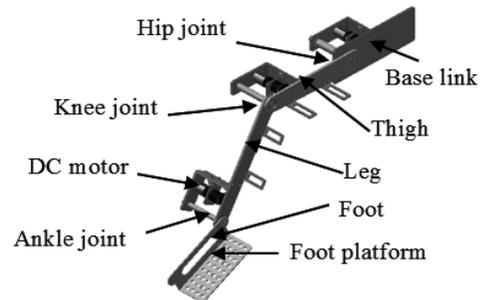


Fig 1. Mechanical structure

Robot's links connected in series by 3 joints, support rehabilitation for hip, knee and joints. The motion of each joint is controlled by an actuator that used a DC motor. Linkage diagram of the robot shown in Fig. 2. A Cartesian coordinate is attached to each link of the robot. Because all of joints axis are parallel to each other, so all the twist angles  $\alpha_i$  and translational distances  $d_i$  are zero.

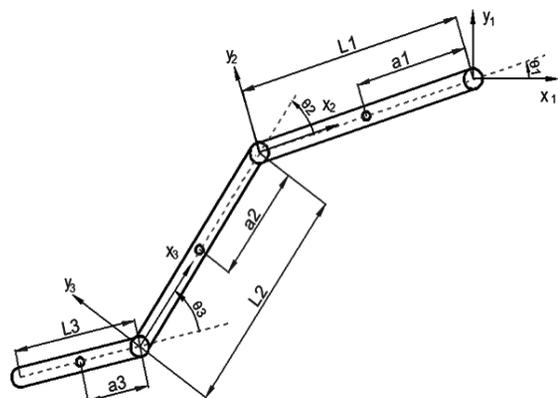


Fig. 2. Linkage diagram

**2.2. Modeling**

According to the mechanical structure defined in this study, mathematical model of the robot composed of kinematic and dynamic model, friction model and motor model. Kinematic and dynamic model describes position of robot and the relationship between the lower limb robot and joint torque; friction model depicts the compensation to servo system for influence of fiction; motor model shows the principle of converting energy from electricity to kinetic energy.

**2.2.1. Kinematic and dynamic model**

There are several methods for kinematic and dynamic modeling. In this study, Denavid-Hartenberg (D-H) and Lagrange equations are chosen to establish the kinematic and dynamic model of robot. For the coordinate systems chosen, the link parameters are given in Table 1.

Table 1. D-H Parameters of Robot

Joint i	$\alpha_i$	$a_i$	$d_i$	$\theta_i$
1	0	$l_1$	0	$\theta_1$
2	0	$l_2$	0	$\theta_2$
3	0	$l_3$	0	$\theta_3$

The D-H transformation matrices are obtained by substituting the D-H parameters into equation (1) and (2) [18].

$${}^{i-1}A_i = \begin{bmatrix} \cos \theta_i & -\sin \theta_i \cos \alpha_i & \sin \theta_i \sin \alpha_i & a_i \cos \theta_i \\ \sin \theta_i & \cos \theta_i \cos \alpha_i & -\cos \theta_i \sin \alpha_i & a_i \sin \theta_i \\ 0 & \sin \alpha_i & \cos \alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$A_i = {}^0A_1 {}^1A_2 \dots {}^{i-1}A_i \quad (2)$$

Where

$a_i$ : offset distance between two adjacent joint axes;  $\alpha_i$ : twist angle between two adjacent joint axes. It is the angle rotate the  $z'_{i-1}$  axis into alignment with the  $z_i$  about the positive the  $x_i$  axis according to right-hand rule ( $z'_{i-1} // z_{i-1}$ , through  $O_i$  point);  $\theta_i$ : is the angle rotate the  $x_{i-1}$  into alignment with the  $x_i$  axis about the positive the  $z_{i-1}$  axis according to right-hand rule;  $d_i$ : is the translational distance between two incident normal of a joint axis.

Substituting D-H parameter in Table 1 into Eqs.(1) and (2), we obtain constant transformation matrices:

$$A_1 = {}^0A_1 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 & l_1 \cos \theta_1 \\ \sin \theta_1 & \cos \theta_1 & 0 & l_1 \sin \theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$A_2 = {}^0A_1 {}^1A_2 = \begin{bmatrix} \cos \theta_{12} & -\sin \theta_{12} & 0 & l_1 \cos \theta_1 + l_2 \cos \theta_{12} \\ \sin \theta_{12} & \cos \theta_{12} & 0 & l_1 \sin \theta_1 + l_2 \sin \theta_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$A_3 = {}^0A_1 {}^1A_2 {}^2A_3 = \begin{bmatrix} \cos \theta_{123} & -\sin \theta_{123} & 0 & l_1 \cos \theta_1 + l_2 \cos \theta_{12} + l_3 \cos \theta_{123} \\ \sin \theta_{123} & \cos \theta_{123} & 0 & l_1 \sin \theta_1 + l_2 \sin \theta_{12} + l_3 \sin \theta_{123} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

with:  $\theta_{12} = \theta_1 + \theta_2$ ,  $\theta_{123} = \theta_1 + \theta_2 + \theta_3$ ,

From (3), (4), (5), we obtain direction cosines matrices of 3 joints:

$$R_1 = \begin{bmatrix} \cos \theta_1 & -\sin \theta_1 & 0 \\ \sin \theta_1 & \cos \theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$R_2 = \begin{bmatrix} \cos \theta_{12} & -\sin \theta_{12} & 0 \\ \sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$R_3 = \begin{bmatrix} \cos \theta_{123} & -\sin \theta_{123} & 0 \\ \sin \theta_{123} & \cos \theta_{123} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (8)$$

Use matrix form of Lagrange equation, the dynamic equation of motion is written matrix form as [12]:

$$M(q)\ddot{q} + V(q, \dot{q}) + G(q) = \tau \quad (9)$$

Where:  $M(q)$  is the manipulator inertia matrix,  $V(q, \dot{q})$  is the velocity coupling vector,  $G(q)$  is the gravitational vector,  $\tau = [\tau_1 \ \tau_2 \ \tau_3]^T$  is the vector of generalized forces and  $q = [\theta_1 \ \theta_2 \ \theta_3]^T$  is vector of generalized Lagrange coordinates.

**Manipulator inertia matrix:** The manipulator inertia matrix is obtained by Eqs. (10) [18]:

$$M = \sum_{i=1}^3 (m_i J_{Ti}^T J_{Ti} + J_{Ri}^T I_i J_{Ri}) = \begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix} \quad (10)$$

Where  $m_i$ : mass of link  $i$ ,  $J_{Ti}$  and  $J_{Ri}$  are link  $i$  Jacobian submatrices and  $I_i$  is the inertia matrix of link  $I$  about its center of mass and expressed in the base link frame. These matrices are obtained by:

$$I_i = R_i {}^iI_i (R_i)^T \quad \text{with} \quad {}^iI_i = \frac{1}{12} m_i l_i^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad (11)$$

$$J_{Ti} = \frac{\partial r_{ci}}{\partial q} \quad \text{and} \quad J_{Ri} = \frac{\partial \omega_i}{\partial \dot{q}}$$

Where  $r_{c_i}$  and  $\omega_i$  is the position vector of the center of mass and the angular velocity vector of link  $i$ . Position vectors of the center of mass  $r_{c_i}$  is obtained from Fig. 2 as (12):

$$r_{c1} = \begin{bmatrix} a_1 \cos \theta_1 \\ a_1 \sin \theta_1 \\ 0 \end{bmatrix}, r_{c2} = \begin{bmatrix} l_1 \cos \theta_1 + a_2 \cos \theta_{12} \\ l_1 \sin \theta_1 + a_2 \sin \theta_{12} \\ 0 \end{bmatrix},$$

$$r_{c3} = \begin{bmatrix} l_1 \cos \theta_1 + l_2 \cos \theta_{12} + a_3 \cos \theta_{123} \\ l_1 \sin \theta_1 + l_2 \sin \theta_{12} + a_3 \sin \theta_{123} \\ 0 \end{bmatrix} \quad (12)$$

The angular velocity vector of link  $i$  is obtained by  $\bar{\omega}_i = \dot{R}_i R_i^T$  we have:

$$\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}, \omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{12} \end{bmatrix} \text{ and } \omega_3 = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{123} \end{bmatrix} \quad (13)$$

with  $\dot{\theta}_{12} = \dot{\theta}_1 + \dot{\theta}_2$ ,  $\dot{\theta}_{123} = \dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_3$

Substituting Eq (11), (12) and (13) into (10), we have elements of  $M(q)$  matrix are:

$$m_{11} = a_1^2 m_1 + (a_2^2 + l_2^2 + 2a_2 l_1 \cos \theta_1) m_2 + (a_3^2 + l_1^2 + l_2^2 + 2l_1 l_2 \cos \theta_2 + 2a_3 l_1 \cos \theta_{23} + 2a_3 l_1 \cos \theta_3) m_2$$

$$m_{12} = m_{21} = (a_2^2 + a_2 l_1 \cos \theta_2) m_2 + (a_3^2 + l_2^2 + 2a_3 l_2 \cos \theta_3 + a_3 l_1 \cos \theta_{23} + l_1 l_2 \cos \theta_2) m_3$$

$$m_{13} = m_{31} = (a_3^2 + a_3 l_1 \cos \theta_{23} + a_3 l_2 \cos \theta_3) m_3;$$

$$m_{22} = a_2^2 m_2 + (a_3^2 + l_2^2 + 2a_3 l_2 \cos \theta_3) m_3;$$

$$m_{23} = m_{32} = (a_3^2 + a_3 l_2 \cos \theta_3) m_3;$$

$$m_{33} = \left( a_3^2 + \frac{1}{12} l_3^2 \right) m_3 \quad (14)$$

**The velocity coupling vector:** The velocity coupling vector  $V(q, \dot{q})$  is obtained by Eqs (15):

$$V = [V_1 \quad V_2 \quad V_3]^T$$

with  $V_i = \sum_{j=1}^3 \sum_{k=1}^3 \left( \frac{\partial m_{ij}}{\partial \theta_k} - \frac{1}{2} \frac{\partial m_{jk}}{\partial \theta_i} \right) \dot{\theta}_j \dot{\theta}_k$  (15)

Substituting  $m_{ij}$ ,  $m_{jk}$  in (14) into (15), we have:  $V_1 = -a_2 l_1 m_2 \sin \theta_1 \dot{\theta}_1^2 - 2(l_1 l_2 m_3 \sin \theta_2 +$

$$a_3 l_1 m_3 \sin \theta_{23}) \dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_3 - 2(a_3 l_1 m_3 \sin \theta_{23} + a_3 l_1 m_3 \sin \theta_3) \dot{\theta}_1 \dot{\theta}_3^2 - (a_2 l_1 m_2 \sin \theta_2 + a_3 l_1 m_3 \sin \theta_{23} - l_1 l_2 m_3 \sin \theta_2) \dot{\theta}_2^3 - (2a_3 l_2 m_3 \sin \theta_3 + a_3 l_1 m_3 \sin \theta_{23}) \dot{\theta}_2 \dot{\theta}_3^2 - a_3 l_1 m_3 \sin \theta_{23} \dot{\theta}_2^2 \dot{\theta}_3 - (a_3 l_1 m_3 \sin \theta_{23} + a_3 l_2 m_3 \sin \theta_3) \dot{\theta}_3^3$$

$$V_2 = (l_1 l_2 m_3 \sin \theta_2 + a_3 l_1 m_3 \sin \theta_{23}) \dot{\theta}_1^2 \dot{\theta}_2 - (2a_3 l_2 m_3 \sin \theta_3 + a_3 l_1 m_3 \sin \theta_{23}) \dot{\theta}_1 \dot{\theta}_3^2 + \frac{3}{2} a_3 l_1 m_3 \sin \theta_{23} \dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_3 - 2a_3 l_2 m_3 \sin \theta_3 \dot{\theta}_2 \dot{\theta}_3^2 - a_3 l_2 m_3 \sin \theta_3 \dot{\theta}_3^3$$

$$V_3 = (a_3 l_1 m_3 \sin \theta_{23} + a_3 l_1 m_3 \sin \theta_3) \dot{\theta}_1^2 \dot{\theta}_3 + 2a_3 l_2 m_3 \sin \theta_3 \dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_3 + a_3 l_2 m_3 \sin \theta_3 \dot{\theta}_2^2 \dot{\theta}_3$$

**The gravitational vector:** The gravitational vector  $G(q) = [G_1 \quad G_2 \quad G_3]^T$  has elements  $G_i$  are obtained by:

$$G_i = -\sum_{j=1}^3 m_j g^T J_{T_j}^i \quad (16)$$

We have:

$$G_1 = m_1 g a_1 \cos \theta_1 + m_2 g (l_1 \cos \theta_1 + a_2 \cos \theta_{12}) + m_3 g (l_1 \cos \theta_1 + l_2 \cos \theta_{12} + a_3 \cos \theta_{123})$$

$$G_2 = m_2 g a_2 \cos \theta_{12} + m_3 g (l_2 \cos \theta_{12} + a_3 \cos \theta_{123})$$

$$G_3 = m_3 g a_3 \cos \theta_{123} \quad (17)$$

Substituting matrix M, vector V and vector G into Eqs. (9), we have three dynamical equations of motion are elements in the vector of generalized forces  $\tau = [\tau_1 \quad \tau_2 \quad \tau_3]^T$ .

### 2.2.2. Friction model

Kinetic and dynamic model of robot are obtained base on linkage diagram in Fig. 2, so the effect of non-rigid such as friction is not taken into account. Meanwhile, friction torque is an important factor in rehabilitation robot control. To obtain a more accurate mathematical description of the robot system, a friction model, including both dynamic friction and steady friction, is needed modeling.

When  $\dot{\theta} \neq 0$  we have dynamic friction torque:

$$T_f = F_c \operatorname{sgn}(\dot{\theta}) + b\dot{\theta} \quad (18)$$

When  $\dot{\theta} = 0$  we have steady friction torque:

$$T_f = \begin{cases} T_m & (|T_m| < F_s) \\ F_s & (|T_m| \geq F_s) \end{cases} \quad (19)$$

Where  $F_c$  is the Coulomb friction force,  $b\dot{\theta}$  is viscous friction,  $F_s$  is static friction and  $T_m$  is the output torques of motors. According to the mechanical structure model, we have:

$$T_s = K_g T_m - T_f \quad (20)$$

With  $K_g$  is the gear of motor.

### 2.2.3. Motor model

Motors are used to drive the joints is a DC motor and gear boxes. We have formula which can be obtained by:

$$T_m = K_m i_m \quad (21)$$

Where  $K_m$  is torque constant of DC motor,  $i_m$  is current of DC motor. According to the Kirchoff law, voltage of DC motor is obtained by:

$$U_m = i_m R + i_m L_m + K_e \omega \quad (22)$$

Where  $K_e$  is the back-EMF constant of DC motor,  $R$  and  $L_m$  is resistance and inductance of DC motor,  $\omega$  is angular velocity. Combining Eqs.(21) and (22)

we have the relationship between  $U_m$  và  $\dot{T}_m$  as:

$$\dot{T}_m = \frac{U_m}{L_m} K_m - \frac{T_m}{L_m} R - \frac{K_e K_m}{L_m} \omega \quad (23)$$

If the current of the motor is measured, we can obtain the motor torque and with the information of the angular velocity we can also control the motor output torque accurately.

**3. Simulation Results**

Based on the dynamic model (9), friction model (18) and motor model (23), a simulation model is built in Matlab/Simulink to check the accuracy and suitability with the physical characteristics of the system. Block diagram of simulation model is shown in Fig. 3. The parameters of kinematic and dynamic model is chosen, while the parameters of motor model are mainly about DC motor's characteristics, obtained from motor datasheets. Table 2 lists all the parameters of the system.

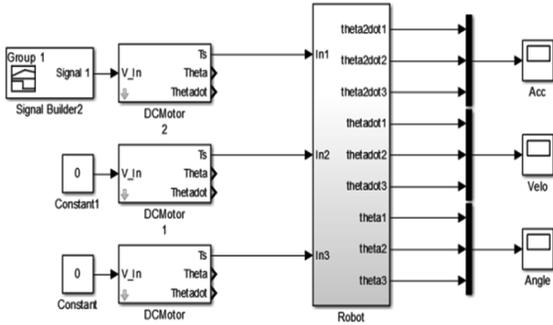


Fig. 3a. Block diagram of simulation model

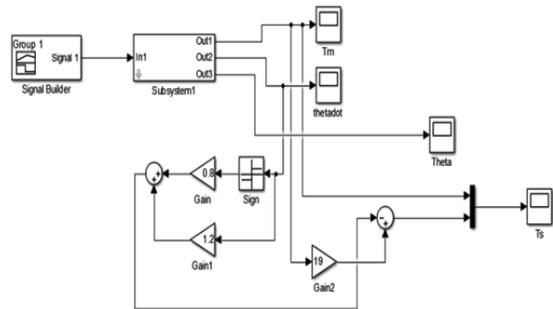


Fig. 3b. Simulation model of motor and friction model

Table 2. Parameter of Robot

Parameter	i = 1	i = 2	i = 3
$m_i$ (kg)	3.2	3.1	0.25
$L_i$ (m)	0.4	0.48	0.33
$a_i$ (m)	0.2	0.24	0.165
$F_s$ (Nm)	0.9	0.9	0.9
$F_c$ (Nm)	0.8	0.8	0.8

$b_i$ (Nm/rad/s)	1.2	1.2	1.2
$K_m$ (Nm/A)	0.08	0.08	0.08
$K_e$ (V/rad/s)	0.287	0.287	0.287
$L_m$ (H)	0.0026	0.0026	0.0026
$R_m$ ( $\Omega$ )	1.2	1.2	1.2
$I_m$ (kg m <sup>2</sup> )	0.000025	0.000025	0.000025
$K_g$	19	19	19

Simulation result of motor and friction model with some values of  $U_m$  : 10V, 5V and 3V are shown in Fig. 6. From this figure, it can be seen that with a value of each input voltage, rotation angular of motor  $\theta_m$  also changes accordingly. It shows that the mathematical description model of the mechanical structure and friction model is accurate and suitable with the motor's physical characteristics.

Fig. 4 and Fig. 5 gives simulation result of the system model. Input voltage  $U_m$  is only sent to motor drive the hip joint, input voltage for other joints is zero.

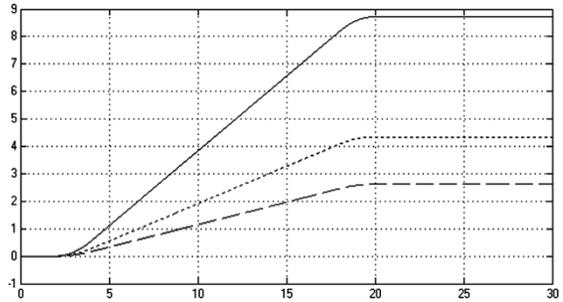


Fig. 4. Simulation result of motor and friction

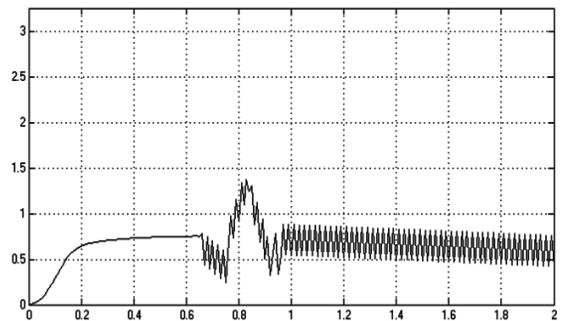


Fig. 5. Simulation result of the system

From Fig. 5, it can be seen that when the voltage signal is applied to the drive motor for the hip joint, the robot's hip joint will rotate, the other joint angles remain unchanged and equal to zero. This is consistent with the physical characteristics of the system and shows the main accuracy of the mathematical model of the system. The fluctuation

of the joint angle in this case is unavoidable because this is an open loop diagram, without control. Therefore, in order to ensure accurate motions and eliminate the rotational oscillations of the robot, it is necessary to have a controller with suitable control algorithms to control the robot according to the requirements and technical specifications.

#### 4. Conclusion

In this study, the mathematic model of 3 DOF rehabilitation robot including kinetic and

dynamic model of linkage, fiction model and motor model has been built. The mechanical structure and kinematic diagram of the robot are identified with 3 joints to assist in restoring leg motional function to patients including hip joints, knee and ankle joints. The simulation results show the accuracy and suitability of the physical characteristics of the mathematical model of the robot. This modeling is important for calculating, manufacturing hardware and developing control algorithms for robots in next studies.

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## MÔ HÌNH TOÁN HỌC VÀ MÔ PHỎNG RÔ BỐT PHỤC HỒI CHỨC NĂNG CHÂN BA BẬC TỰ DO CHO BỆNH NHÂN SAU CHẤN THƯƠNG

### Tóm tắt:

*Đột quỵ là một bệnh có nguy cơ cao gây tàn tật cho người mắc bệnh. Bệnh nhân đột quỵ có thể cải thiện chức năng vận động nhờ chương trình tập luyện phục hồi chức năng. Nhờ tiến bộ của khoa học - công nghệ, các bài tập phục hồi chức năng có thể thực hiện nhờ sự trợ giúp của Robot. Bài báo này trình bày kết quả việc xây dựng các mô tả toán học của một Robot phục hồi chức năng chân có 3 bậc tự do (3 DOF), bao gồm: mô hình động học và động lực học, mô hình ma sát và mô hình động cơ. Đồng thời, xây dựng mô hình mô phỏng trên phần mềm MATLAB/Simulink để tìm hiểu đặc tính động học của hệ robot. Các kết quả mô phỏng cho thấy tính chính xác và phù hợp với các đặc tính vật lý của mô hình hệ thống. Các kết quả này sẽ được sử dụng cho việc tính toán thiết kế, chế tạo phần cứng và xây dựng thuật toán điều khiển robot sẽ được tiếp tục nghiên cứu và trình bày trong các công bố tiếp theo.*

**Từ khóa:** *chức năng vận động, robot phục hồi chức năng, mô hình toán học, mô hình động lực học, mô phỏng.*