



STUDY ON OSCILATION OF HIGH – RISE BUILDINGS SUBJECTED TO COERCIVE FORCES

Nguyen Huu Dinh¹, Le Thi Mai¹, Khong Doan Dien²

1 Vietnam Maritime University

2 Hoa Binh University

Received: 10/12/2019

Revised: 10/3/2020

Accepted for publication: 22/3/2020

Abstract:

This paper studies two oscillation moduls. The first problem studies the oscillation of a foundation of a building. The foundation of the building is considered to be a flat block resting on the springs in two vertical and horizontal directions subjected to vertical coercive forces $F(t)$. The second problem investigates the oscillation of a two-story building subjected to the force $F(t)$ acting on the top floor of the building.

Keywords: *Oscilation of high – rise buildings.*

1. Introduction

Oscilation of high – rise buildings and The effects of oscillation on high-rise buildings are a matter of much research. We know that under the influence of wind force or earthquake. High-rise buildings may fluctuate. The vibrations of a building damage the structure of the building or move objects in the building. The strong vibration of a building can cause it to collapse. Therefore the oscillation of high-rise buildings the author is interested in researching in this paper.

2. Build some oscillating mo dels

2.1. First oscillator model

a. The first problem

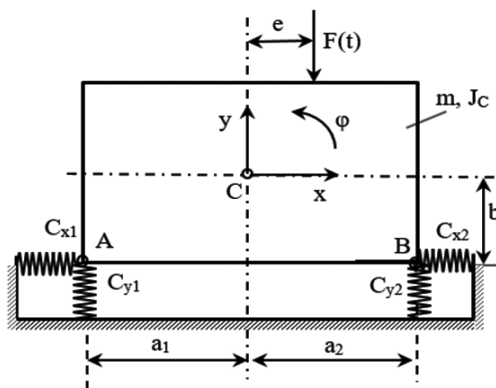


Figure 1a

The model is the foundation of the building

that is considered to be flat. It has m weight on the springs. The springs are vertical and horizontal. It is subjected to vertical $F(t)$ forces (Figure 1a). Establish a small oscillation equation of the foundation around the static equilibrium position. Assume that the foundations only fluctuate in the drawing plane. Figure 1a the unformed springs.

b. Set up the oscillation equation of the first model

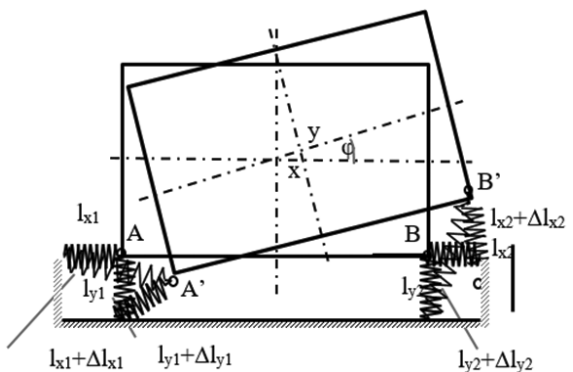


Figure 1b

Kinetic energy expression

$$T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2) + \frac{1}{2} J_C \dot{\phi}^2$$

When the building’s foundation hovers in the plane. The long deformation of the springs is Δl_{x1} , Δl_{x2} , Δl_{y1} , Δl_{y2} (Figure 1b). Potential energy of the system is

$$\Pi = mgy + \frac{1}{2} (c_{x1} \Delta l_{x1}^2 + c_{x2} \Delta l_{x2}^2 + c_{y1} \Delta l_{y1}^2 + c_{y2} \Delta l_{y2}^2)$$

We denote the displacement components of point A (from A to A') are u_1 and v_1 , The displacement components of point B (from B to B') are u_2 and v_2 . According to Figure 1b. we have

$$\begin{aligned} u_1 &= x + a_1(1 - \cos \varphi) + b \sin \varphi; \\ u_2 &= x - a_2(1 - \cos \varphi) + b \sin \varphi; \\ v_1 &= y - a_1 \sin \varphi + b(1 - \cos \varphi); \\ v_2 &= y + a_2 \sin \varphi + b(1 - \cos \varphi) \end{aligned}$$

Apply the cosin theorem. we calculate

$$\begin{aligned} \Delta l_{x1}^2 &= l_{x1}^2 \left[\sqrt{1 + (u_1^2 + v_1^2 + 2l_{x1}u_1/l_{x1}^2 - 1)} \right]^2 \\ \Delta l_{y1}^2 &= l_{y1}^2 \left[\sqrt{1 + (u_1^2 + v_1^2 - 2l_{y1}v_1/l_{y1}^2 - 1)} \right]^2 \\ \Delta l_{x2}^2 &= l_{x2}^2 \left[\sqrt{1 + (u_2^2 + v_2^2 - 2l_{x2}u_2/l_{x2}^2 - 1)} \right]^2 \\ \Delta l_{y2}^2 &= l_{y2}^2 \left[\sqrt{1 + (u_2^2 + v_2^2 + 2l_{y2}v_2/l_{y2}^2 - 1)} \right]^2 \end{aligned}$$

The quantities $l_{x1}, l_{x2}, l_{y1}, l_{y2}$ is the length of the spring when the spring has not been deformed.

because we consider the small vibration around the equilibrium position. The following approximations should be used.

$$\Delta l_{x1}^2 \approx u_1^2; \Delta l_{x2}^2 \approx u_2^2; \Delta l_{y1}^2 \approx v_1^2; \Delta l_{y2}^2 \approx v_2^2$$

Due to the small φ angle, we have an approximation $\sin \varphi \approx \varphi, \cos \varphi \approx 1$. I can calculate

$$\begin{aligned} \Delta l_{x1}^2 &\approx (x + b\varphi)^2; \Delta l_{y1}^2 \approx (y - a_1\varphi)^2 \\ \Delta l_{x2}^2 &\approx (x + b\varphi)^2; \Delta l_{y2}^2 \approx (y + a_2\varphi)^2 \end{aligned}$$

Thus the approximate expression of the potential energy has a form

$$\Pi = \frac{1}{2} \left[c_{x1}(x + b\varphi)^2 + c_{x2}(x + b\varphi)^2 + c_{y1}(y - a_1\varphi)^2 + c_{y2}(y + a_2\varphi)^2 \right] + mgy$$

To determine the extrapolation force corresponding to the force $F(t)$.

$$\delta A = -F(t)\delta y - F(t)e\delta\varphi$$

The extrapolation of forces has no potential energy

$$Q_x^* = 0; Q_y^* = -F(t); Q_\varphi^* = -F(t)e$$

Plug the above expressions in Equation Lagrange II as follows

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} = -\frac{\partial \Pi}{\partial q_i} + Q_i^*$$

We get the equation of the foundation oscillation of the form

$$M\ddot{q} + Cq = f(t)$$

With the following components

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_C \end{bmatrix}; q = \begin{bmatrix} x \\ y \\ \varphi \end{bmatrix}; f = - \begin{bmatrix} 0 \\ mg + F \\ eF \end{bmatrix}$$

$$C = \begin{bmatrix} c_{x1} + c_{x2} & 0 \\ 0 & c_{y1} + c_{y2} \\ (c_{x1} + c_{x2})b & a_2c_{y2} - a_1c_{y1} \\ & (c_{x1} + c_{x2})b \\ & a_2c_{y2} - a_1c_{y1} \\ [(c_{x1} + c_{x2})b^2 + a_1^2c_{y1} + a_2^2c_{y2}] \end{bmatrix}$$

2.2. Second oscillator model

a. The second problem

A two-story building is considered as a two-weight object $m_1 = 2m_2 = 2m$. It oscillates horizontally. Stiffness of the wall is $c_1 = 2c_2 = 2c$. Indicate the force of the wind against the top floor of a building in the form $F(t) = F_0 \sin \Omega t$. Study the law of oscillation of the system.

b. Set up the oscillation equation of the second model

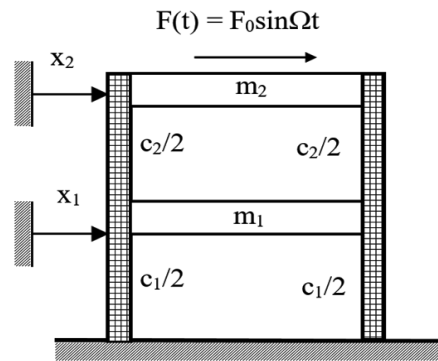


Figure 2

Select the coordinates of the system (x_1, x_2) , With x_1 and x_2 the horizontal displacement of the first and second floor. the corresponding mass is m_1 and m_2 compared to the vertical equilibrium position.

Kinetic energy and Potential energy of the system is

$$\begin{aligned} T &= \frac{1}{2} 2m\dot{x}_1^2 + \frac{1}{2} m\dot{x}_2^2 \\ \Pi &= \frac{1}{2} 2cx_1^2 + \frac{1}{2} c(x_2 - x_1)^2 \end{aligned}$$

Instead of the Lagrange II equation, we derive the oscillating equation of the system

$$\begin{cases} 2m\ddot{x}_1 + 3cx_1 - cx_2 = 0 \\ m\ddot{x}_2 - cx_1 + cx_2 = F_0 \sin \Omega t \end{cases}$$

- Frequency and specific form

Frequency equation

$$|C - \omega^2 M| = 0$$

$$\begin{vmatrix} 3c - 2m\omega^2 & -c \\ -c & c - m\omega^2 \end{vmatrix} = 0$$

Put $\lambda = \omega^2 \frac{m}{c}$ We obtain the equation

$$\lambda^2 - \frac{5}{2}\lambda + 1 = 0$$

The solution of the above equation is

$$\lambda_1 = 0,5; \lambda_2 = 2$$

Correspondingly we calculated

$$\omega_1^2 = 0,5 \frac{c}{m}, \quad v_1 = [0,5 \quad 1]^T;$$

$$\omega_2^2 = 2 \frac{c}{m}; \quad v_2 = [-1 \quad 1]^T$$

Distinct matrices are

$$V = \begin{bmatrix} 0,5 & -1 \\ 1 & 1 \end{bmatrix}$$

- Differential equations in the form of major coordinates

Change $x = Vp$, We have

$$V^T M V = m \begin{bmatrix} 1,5 & 0 \\ 0 & 3,0 \end{bmatrix}, \quad V^T C V = c \begin{bmatrix} 0,75 & 0 \\ 0 & 6 \end{bmatrix},$$

$$V^T f = \begin{bmatrix} 1 \\ 1 \end{bmatrix} F_0 \sin \Omega t$$

Differential equations in the form of major coordinates

$$\begin{cases} 1,5m\ddot{p}_1 + 0,75 \cdot c \cdot p_1 = F_0 \sin \Omega t \\ 3m\ddot{p}_2 + 6 \cdot c \cdot p_2 = F_0 \sin \Omega t \end{cases}$$

or

$$\begin{cases} m\ddot{p}_1 + \omega_1^2 p_1 = \frac{2F_0}{3m} \sin \Omega t \\ m\ddot{p}_2 + \omega_2^2 p_2 = \frac{F_0}{3m} \sin \Omega t \end{cases}$$

The solution of the above equation is

$$\begin{cases} p_1(t) = c_{11} \cos \omega_1 t + c_{12} \sin \omega_1 t + \frac{2F_0}{3m} \cdot \frac{1}{\omega_1^2 - \Omega^2} \sin \Omega t \\ p_2(t) = c_{21} \cos \omega_2 t + c_{22} \sin \omega_2 t + \frac{F_0}{3m} \cdot \frac{1}{\omega_2^2 - \Omega^2} \sin \Omega t \end{cases}$$

Returns to the original coordinates, Oscillation of system is $x = v_1 p_1 + v_2 p_2$. We get the following result.

$$x_1 = 0,5(c_{11} \cos \omega_1 t + c_{12} \sin \omega_1 t) + \frac{2F_0}{3m} \cdot \frac{\sin \Omega t}{\omega_1^2 - \Omega^2} - (c_{21} \cos \omega_2 t + c_{22} \sin \omega_2 t) - \frac{F_0}{3m} \cdot \frac{\sin \Omega t}{\omega_2^2 - \Omega^2}$$

$$x_2 = (c_{11} \cos \omega_1 t + c_{12} \sin \omega_1 t) + \frac{2F_0}{3m} \cdot \frac{\sin \Omega t}{\omega_1^2 - \Omega^2} + (c_{21} \cos \omega_2 t + c_{22} \sin \omega_2 t) + \frac{F_0}{3m} \cdot \frac{\sin \Omega t}{\omega_2^2 - \Omega^2}$$

3. Horizontal vibration results of a two-story building

With force has a form $F(t) = F_0 \sin \Omega t$

Table 1. Datasheets of the building

m_1	262,69x10 ³ kg
m_2	262,69x10 ³ kg
c_1	2x88,56x10 ⁶ N/m
c_2	88,56x10 ⁶ N/m
F_0	50x10 ⁶ N
Ω	100π

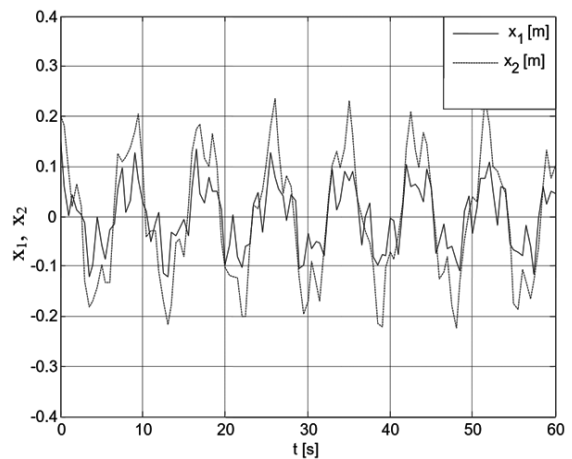


Figure 3. Oscillating graph of a two-story building

4. Conclusion

The two oscillating models are. Oscillation of foundation is considered as flat block and Horizontal fluctuations of a two-story building are exerted by the force $F(t)$ acting on the top floor of the building. For two-storey building, the author has found analytic solution. The author then uses Matlab software to simulate the number.

References

- [1]. Nguyễn Văn Khang, Thái Mạnh Cầu, Vũ Văn Khiêm, Nguyễn Nhật Lê, *Bài tập dao động kỹ thuật*, NXB Khoa học và kỹ thuật, Hà Nội, 2002.
- [2]. Nguyễn Văn Khang, *Dao động kỹ thuật*, NXB Khoa học và kỹ thuật, Hà Nội, 1998.
- [3]. Nguyễn Văn Khang, *Động lực học hệ nhiều vật*, NXB Khoa học và kỹ thuật, Hà Nội, 2007.
- [4]. Đỗ Sanh, Nguyễn Văn Đình, Nguyễn Nhật Lê, *Bài tập cơ học Tập 1 (in lần thứ 16)*, NXB Giáo dục Việt Nam, Hà Nội, 2011.
- [5]. Đỗ Sanh, Lê Doãn Hồng, *Bài tập cơ học Tập 2 (in lần thứ 13)*, NXB Giáo dục Việt Nam, Hà Nội, 2011.
- [6]. Hoàng Mạnh Cường, Nguyễn Hữu Đình, Phạm Thị Thúy, *Cơ học lý thuyết*, NXB Hàng Hải, Hải Phòng, 2018.
- [7]. Nguyễn Hữu Đình, Vũ Xuân Trường, Nghiên cứu ảnh hưởng của gió, động đất tới dao động của tòa nhà cao tầng, *Tạp chí Khoa học và Công nghệ*, Trường Đại học Sư phạm kỹ thuật Hưng Yên, Số 23/Tháng 9-2019.

NGHIÊN CỨU DAO ĐỘNG CỦA TÒA NHÀ CAO TẦNG CHỊU TÁC DỤNG CỦA LỰC CƯỜNG BỨC

Tóm tắt:

Bài báo này nghiên cứu hai mô hình dao động. Bài toán thứ nhất nghiên cứu dao động của một móng nhà được coi là một khối phẳng tựa trên các lò xo theo hai phương thẳng đứng và nằm ngang dưới tác dụng của lực $F(t)$ theo phương thẳng đứng. Bài toán thứ hai nghiên cứu dao động theo phương ngang của tòa nhà hai tầng dưới tác dụng của lực $F(t)$ tác dụng vào tầng trên cùng của tòa nhà.

Từ khóa: *Dao động của tòa nhà cao tầng.*