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STUDY ON REDUCING TORSIONAL VIBRATION USING MULTI DYNAMIC VIBRATION ABSORBER ATTACHED SHAFT OF MACHINE

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Abstract :

This paper presented a method to reduce the torsional vibration of a shaft system with dynamic vibration absorber (DVA). A theoretical method was introduced to determine optimal parameters of the DVA, which included spring stiffness, viscous coefficient of damper, mass moment of inertia of the absorber, number, and radial position of springs and dampers. First, system equations of motion of the shaft and the DVA were elaborated using Finite Element method (FEM) and solved by Runge-Kutta algorithm to find the torsional vibration response. Then, the Taguchi method was applied for the multivariable optimization problem. By using the Taguchi method, the DVA optimal parameters were identified with objective functions of torsional vibration duration and amplitude. Analysis of variance (ANOVA) was then carried out to evaluate the contribution percentage of each parameter on the shaft vibration response. The obtained results showed that the radial position of spring was the most influential factor on vibration of the shaft. DVA with optimized parameters remarkably reduced the torsional vibration in the system.

Keywords : *dynamic vibration absorber, torsional vibration, optimal design, FEM, Taguchi method.*

1. Introduction

The torsional vibration is particularly harmful to machine parts and machines. There have been many scientists focusing on this research. The first scientists to open the research fields to reduce the vibration in a passive way using TMD device (Tuned Mass Damper) are such as Frahm, Den Hartog, ... In 1909, Frahm proposed the classic TMD model using springs only. Later, Den Hartog discovered that just using the spring in the model would not effectively reduce the oscillation over a wide range of resonant frequencies. Den Hartog recommends installing parallel a viscous with springs [2], [3], who was the father of the two fixed point method - one of the classic analytical methods in oscillation analysis. Later, it was developed and perfected by Brock [4] with an optimal analytic solution in a simple form.

The steady state behavior of torsional vibration used DVA was often considered in most studies [5-11]. XT Vu [12] determined the optimal parameter of the DVA attached the MDOF shaft system by developing algorithms two fixed points.

This paper presents a theoretical method for determining optimal parameters of a DVA, which is used to reduce vibration of a shaft under time-varying torsional moment. The equations of motion for the shaft-DVA system are created using FEM and solved using Runge-Kutta method to determine vibration response of the shaft. Orthogonal design based on the Taguchi method [13] is then applied to identify the optimal parameters of the DVA with objectives of torsional vibration amplitude and duration. The influence of each design parameter on overall vibration of the shaft is characterized using an analysis of variance method. Finally, vibration behaviors of the shaft with and without optimal parameters are evaluated to show the effectiveness of the presented method.

2. Mathematical model of the shaft and DVA system

Torsional vibration will be induced in the shaft, which is the relative twisting angle between

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the shaft ends defined *by* $\theta=\varphi_N-\varphi_1$, where φ_1 and φ_N are the rotational angle of left and right ends of the shaft, respectively. Figure 2 shows the DVA model, which consists of sets of linear springs and dampers. The stiffness and the viscous coefficient of the spring and damper are k_a and c_a, respectively. n_a represents the number of the spring and damper sets. e_1 and e_2 indicate the radial position of spring and damper, respectively. ψ is the rotation angle at the shaft end, which is connected to the absorber of radius r. By introducing a relative rotation angle between the DVA and the shaft end (γ), absolute rotation angle of the DVA (φ_a). The model of shaft with N elements and DVA is shown in Figure 3. An element is connected to lateral ones via nodes. The mass moment of inertia and stiffness of a shaft element are J_{si} and k_{si}, respectively (i = 1,…,N).

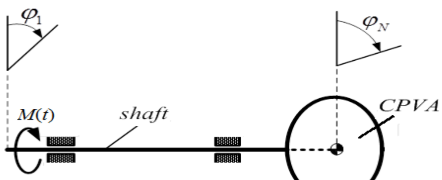

Figure 1. Model of the shaft-DVA system

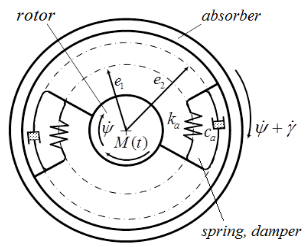

Figure 2. Model of the DVA

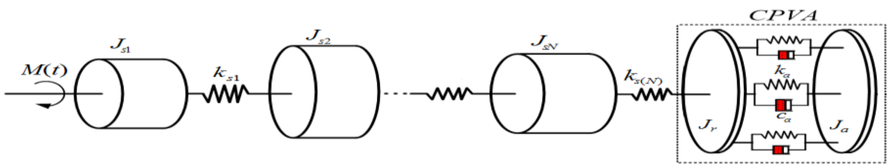

Figure 3. *FEM model of the shaft - DVA system*

All the shaft elements and the DVA are now assembled to give the complete equations of motion for the system, which are described by

$$\boldsymbol{M}\ddot{\boldsymbol{q}}+\boldsymbol{C}\ddot{\boldsymbol{q}}+\boldsymbol{K}\boldsymbol{q}=\boldsymbol{F} \tag{1}$$

where **M**, **C** and **K** are the mass, damping and stiffness matrices of the system, respectively. Those matrices are given as following

$$\boldsymbol{M}=\begin{bmatrix} J_{s1} & 0 & 0 & 0 & 0 & 0 \\ 0 & J_{s2} & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & J_{sN} & 0 & 0 \\ 0 & 0 & 0 & 0 & J_r+J_a & J_a \\ 0 & 0 & 0 & 0 & J_a & J_a \end{bmatrix}_{(N+2)} \tag{2}$$

$$\boldsymbol{C}=n_a c_a e_2^2\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{(N+2)} \tag{3}$$

$$\boldsymbol{K}=\begin{bmatrix} k_{s1} & -k_{s1} & \dots & \dots & \dots & 0 \\ -k_{s1} & k_{s1}+k_{s2} & k_{s2} & \dots & \dots & 0 \\ \dots & -k_{s2} & k_{s2} & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & n_a k_a e_1^2 \end{bmatrix}_{(N+2)} \tag{4}$$

$$\boldsymbol{F}=\{-M_1 \quad 0 \quad \dots \quad M_N - M_N \quad 0\}^T \tag{5}$$

$$\boldsymbol{q}=\{\varphi_1 \quad \varphi_2 \quad \dots \quad \varphi_N \psi \quad \gamma\}^T \tag{6}$$

By solving the equations of motion (1) using Runge-Kutta method, the torsional vibration of the shaft is then found and calculated by

$$\theta=\sum_{i=1}^{N}\theta_i \tag{7}$$

To solve the system equations of motion, Maple software was used in this study.

3. The Optimal design of DVAs using Taguchi Method

In this section, an optimization design for the DVA's parameters based on the Taguchi method will be presented [9]. In this study, objective functions of the optimization design are torsional vibration amplitude and duration. Six design parameters will be investigated including the stiffness of spring (k_a), the viscous coefficient of damper (c_a), the mass moment of inertia of the DVA (J_a), the number of spring and damper sets (n_a) and radial position of spring (e_1) and damper (e_2). The design parameters are introduced in following forms

$$f_1 = \frac{e_1}{r};\ f_2 = \frac{e_2}{r};\ f_3 = \frac{J_a}{J_s}; \quad (8)$$

$$f_4 = n_a; f_5 = \frac{k_a}{k_s};\ f_6 = \frac{c_a}{k_s}$$

In this paper, the number of design parameters is six and the level of each parameter is chosen as five. Therefore, total 25 trials are selected for the optimal design based on the Taguchi method (L_{25} array table). From the level of six parameters in 25 trials, Taguchi proposed a combination scheme for the set of trial parameters to investigate the entire parameter space with a small number of observations. The obtained results are then transformed into a signal-to-noise (S/N) ratio [9].

In order to determine the optimal parameters of the DVA, the S/N ratio will be determined using commercial statistical software package Minitab.

4. Parametric Study

In this section, the above theoretical analysis will be applied for a sample shaft-DVA system. The numerical results of the optimization design are then obtained and discussed.

A sample shaft and a rotor are introduced whose parameters are shown in Table 1 including outer diameter (d_{si}) and length (L_{si}) of each element. Figure 4 shows the schematic of the shaft and rotor. The corresponding torsional stiffness and mass moment of inertia of a shaft element can be calculated using following formulae

$$k_{si} = \frac{0.1Gd_{si}^4}{L_{si}};\ J_{si} = \frac{m_{si}d_{si}^2}{8} \quad (9)$$

where m_{si} and G are mass of the shaft element and shear modulus of material, respectively. In this study, G is selected equal to 8.1010 N/m^2. The equivalent stiffness (k_s) and mass moment of inertia (J_s) of the shaft are calculated by, respectively

$$\frac{1}{k_s} = \sum_{i=1}^{N} \frac{1}{k_{si}}\ ;\ J_s = \sum_{i=1}^{N} J_{si} \quad (10)$$

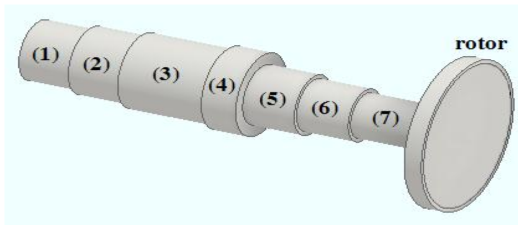

Figure 4. *Schematic of the shaft and rotor*

From the level of parameters shown in Table 2, a combination scheme for the set of trial parameters are obtained as shown in Table 3 and Table 4.

Table 1. Shaft and rotor parameters

Parameter	d_{si} (m)	L_{si} (m)	Parameter	d_{si} (m)	L_{si} (m)
Element (1)	0.040	0.03	Element (5)	0.040	0.03
Element (2)	0.045	0.03	Element (6)	0.035	0.03
Element (3)	0.050	0.05	Element (7)	0.030	0.05
Element (4)	0.055	0.02	Rotor	0.100	0.02

Table 2. Parameter level

Parameter	Level				
	1	2	3	4	5
f_1	0.2	0.4	0.6	0.8	1.0
f_2	0.2	0.4	0.6	0.8	1.0
f_3	0.02	0.04	0.06	0.08	0.1
f_4	2	4	6	8	10
f_5	0.001	0.025	0.128	0.404	0.987

ISSN 2354-0575

f_6	1.377E-06	1.885E-06	1.953E-06	2.242E-06	3.337E-06

Table 3. Layout of the trials using an L25 orthogonal array proposed by Taguchi

Trial no.	Level of parameter					
	f_1	f_2	f_3	f_4	f_5	f_6
1	1	1	1	1	1	1
…						
25	5	5	4	3	2	1

Table 4. Values of parameters according to the combination scheme in Table 4

Trial no.	Value of parameter					
	f_1	f_2	f_3	f_4	f_5	f_6
1	0.2	0.2	0.02	2	0.001	1.377E-06
…	…	…	…	…	…	…
24	1.0	0.8	0.06	4	0.001	3.337E-06
25	1.0	1.0	0.08	6	0.025	1.377E-06

5. Result and Discussion

Figure 5 shows the *S/N* ratios for all trials calculated. It is seen that the trial #24 provides the highest *S/N* ratio. Thus the parameters of trial #24 are expected to be optimal parameters for the DVA.

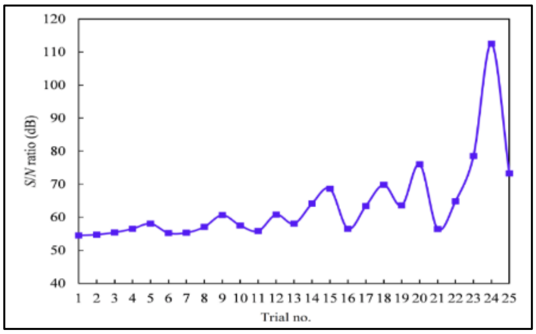

Figure 5. S/N ratio of the trials

Figure 6 further displays the *S/N* ratio response in which mean of *S/N* ratio obtained for all levels of parameters f_i ($i = 1,\ldots, 6$) are derived. From Figure 6, individual optimal level for each parameter is obtained and summarized in Table 5, along with the corresponding parameter value.

The optimal levels in Table 5 match with those of trial #24 shown in Table 4. Therefore, parameters in trial #24 are optimal in this design, which are also in agreement with the results in Figure 5. Tables 6 shows the mean S/N ratios determined for all the levels, Delta and Rank. According to the Taguchi method, the statistic "Delta" defined as the difference between the maximum and minimum mean responses is used to determine the most influencing factor. The "Rank" in Table 6 indicates the rank of each Delta, where the first rank corresponds to the largest Delta. According to this table, f1 is the most influential parameter on the shaft vibration. Table 7 further shows the results of ANOVA in which percentage of contribution factors for each parameter are demonstrated. It is observed that a good agreement is achieved between the results in this table and Table 7's.

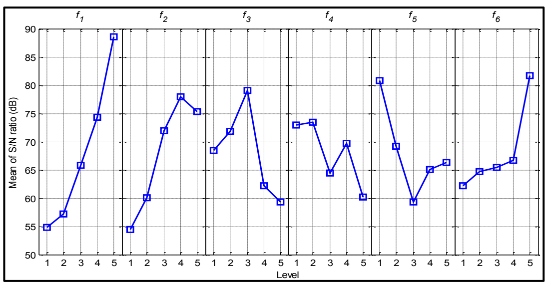

Figure 6. Mean of S/N ratios of the parameters

In general, spring position parameter (f_1) has a maximum contribution with 36.85 %, subsequently to damper position parameter (f_2) with 20.91 %. The spring stiffness, mass moment of inertia and viscous coefficient parameters (f_5, f_3 and f_6) show an approximate contribution of 12.29 %, 11.91 % and 11.66 %, respectively.

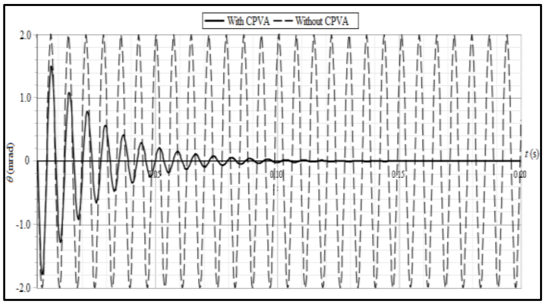

Figure 7. Torsional vibration of shaft without DVA and DVA of the trial #24

Finally, the number of spring and damper sets shows a least influence with a contribution percentage of 6.38%. In this section, the optimal parameters of trial #24 are selected for simulation of shaft torsional vibration responses. For comparison, vibrations of the shaft with parameters of other trials and without DVA are also calculated.

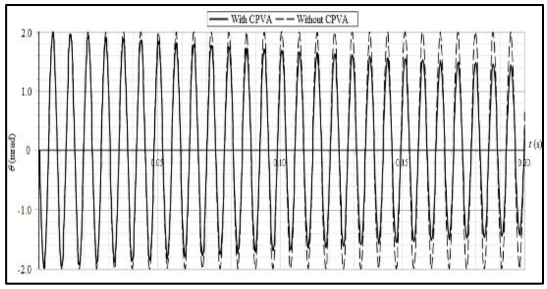

Figure 8. Torsional vibration of shaft without DVA and DVA of the trial #4

Figures 7-8 display the torsional vibrations of the shaft assembled with DVA of trials #24 and #4, along with the shaft without DVA, respectively. Figures 7 shows that trial #24 significantly reduces both the required time for vibration cancelation. It is clear that the vibration reduction effectivenss of the optimal design parameters are remarkable.

Table 5. Optimum level and value for each parameter

Parameter	f_1	f_2	f_3	f_4	f_5	f_6
Optimal level	5	4	3	2	1	5
Value	1	0.8	0.06	4	0.001	3.337E-06

Table 6. Response table for S/N ratio

Level	**Mean *S/N* ratio (dB)**					
	f_1	f_2	f_3	f_4	f_5	f_6
1	54.78	.42	40	72.96	80.84	62.22
2	57.20	0.06	71.77	73.49	69.17	64.68
3	65.80	1.90	79.05	64.38	59.27	65.42
4	74.29	7.99	62.13	69.67	65.08	66.67
5	8.62	5.32	59.34	60.18	66.32	81.70
Delta	3.83	3.58	19.71	13.31	21.57	19.48
Rank	1	2	4	6	3	5

Table 7. Results of ANOVA

Parameters	**Error degrees of freedom**	**Sum of squared deviations (SS_d)**	**% of contribution**	**Rank**
f_1	4	3803.4	36.85%	1
f_2	4	2158.3	20.91%	2
f_3	4	1229.1	11.91%	4
f_4	4	658.4	6.38%	6
f_5	4	1268.5	12.29%	3
f_6	4	1202.9	11.66%	5
Total	24	10320.6	100.00%	

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5. Conclusion.

An optimal design using the Taguchi method for determining the optimal parameters of a DVA has been presented in this study. Design optimization is implemented for multiple parameters of the DVA with the objectives of vibration cancellation time and amplitude. The system equations of motion of the torsion shaft is determined by FEM. The calculation results show that design optimization of DVA is of great importance to obtain the desired effectiveness of torsional vibration reduction. Calculation of torsional vibration with the obtained optimal parameters clearly magnifies the effectiveness of the presented method, which can be successfully applied for optimization of other mechanical systems.

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NGHIÊN CỨU GIẢM DAO ĐỘNG XOẮN CHO TRỤC MÁY SỬ DỤNG KẾT HỢP NHIỀU BỘ TẮT CHẤN ĐỘNG LỰC

Tóm tắt:

Bài báo trình bày một phương pháp để giảm dao động xoắn của hệ thống trục với bộ hấp thụ dao động (DVA). Một phương pháp lý thuyết đã được giới thiệu để xác định các thông số tối ưu của DVA, bao gồm độ cứng của lò xo, hệ số giảm chấn nhớt, mômen quán tính khối lượng của chất hấp thụ, số lượng và vị trí lắp của lò xo và dầu giảm chấn. Đầu tiên, các phương trình dao động của hệ thống trục và DVA được xây dựng bằng phương pháp phần tử hữu hạn (FEM) và được giải bằng thuật toán Runge-Kutta để tìm đáp ứng dao động xoắn. Sau đó, phương pháp Taguchi được áp dụng cho bài toán tối ưu hóa đa biến. Bằng cách sử dụng phương pháp Taguchi, các tham số tối ưu DVA đã được xác định với các hàm mục tiêu về thời gian và biên độ dao động xoắn. Phân tích phương sai (ANOVA) sau đó được thực hiện để đánh giá tỷ lệ phần trăm ảnh hưởng của từng thông số trên đến hiệu quả giảm dao động xoắn. Kết quả thu được cho thấy vị trí lắp của lò xo là yếu tố ảnh hưởng lớn nhất đến độ rung của trục. DVA với các thông số tối ưu hóa làm giảm đáng kể dao động xoắn trong hệ thống.

***Từ khóa**: hấp thụ dao động, dao động xoắn, thiết kế tối ưu, FEM, phương pháp Taguchi.*