ISSN 2354-0575

A NEW APPROACH ON OPTIMAL DESIGN OF CENTRIFUFAL PENDULUM VIBRATION ABSORBERS FOR SHAFT MODEL

Vu Xuan-Truong[1,2], Khong Doan-Dien[1], Nguyen Duy-Chinh[1]
1 Hung Yen University of Technology and Education
2 Institute of Mechanics, VAST

Received: 09/06/2016
Revised: 05/08/2016
Accepted for publication: 12/09/2016

Abtract:

This paper presented a combined theoretical and experimental design methods to determine optimal parameters of a centrifugal pendulum vibration absorber (CPVAs), such as spring stiffness, damper oil viscosity, moment of uinertia of the system components, number of absorbers and assembly position. First, the system of equations of motion of the CPVA was established and solved using Finite Element method (FEM) and Runge-Kutta algorithm (RKA). Then, the optimal design based on Taguchi method was carried out to find optimal parameters of the CPVA with consideration for the torsional vibration duration and stability criterion. The numerical results showed that torsional vibration of the CPVA is remarkably reduced with using the derived optimal parameters.

Keywords: *Centrifugal pendulum vibration absorber, torsional vibration, optimal parameters, FEM, Runge-Kutta algorithm, Taguchi method.*

1. Introduction

Torsional vibrations of rotating systems are induced primarily by torques transmitted to a rotor from forces applied to attach components. Excessive torsional vibrations in mechanical systems with rotating mechanical components result in noise, excessive stresses, or fatigue failure. Therefore, they should be suppressed or controlled immediately to ensure system's reliability. The passive torsional vibration control has been frequently applied due to its simplicity, large range of torsional moment, and acceptable efficiency. Among the passive control techniques, centrifugal pendulum vibration absorber (CPVA) is one of the most widely used methods, which can be found in various heavy-duty machineries, for example, helicopter rotors, radial aircraft engines and combustion engines. This study deals with design optimization of a CPVA parameters for minimize torsional vibration of rotating shafts.

A CPVA consists of masses mounted on the rotor in the manner so that they can freely move along prescribed paths relative to the rotating system. Motions of the masses are used to counteract the applied fluctuating torque, thus reducing torsional vibration of the rotor. The first design of CPVA was introduced by Kuzbach, which comprised of masses moving in U-shaped grooves filled with fluid. In 1929, Carter developed a roll form CPVA for use in diesel engines [12]. Latter, CPVAs with different designs were introduced by using bifilar and roll form pendulums incorporating with different path and damping parameters. Taylor [13] proposed the CPVA for use in geared radial aircraft engines with varying speed condition. In his study, a pendulous weight was constructed so that the restoring force varied with speed. Sarazin introduced the CPVA, which included a compact design pendulum with rollers applying for aircraft engines with increased power output. Until early 1980, the design of almost CPVAs employed the simple circular path for the absorbers (Alsuwaiyan and Shaw). Latter, with increasing demand for torsional vibration reduction effectiveness, the design of CPVA using different specified paths was considered. The use of non-circular paths was found to be quite effective in reducing the level of torsional oscillations over a large torque range [Denman (1992), Madden (1980,[5])]. Many authors [Shaw et al. (1997-2006,[1-7,11,14)] have utilized various specified paths for the CPVA absorber, such as epicycloidal, sub-harmonic epicycloidal, tautochronic and general-paths. Nowadays, the studies on CPVA are increasing interest due to the continuous increase of pressures for higher fuel efficiency, lower emissions, or quieter operation of rotating systems. Swank [17] et-al. (2011) used the combination of both tuned mass absorbers and centrifugal pendulum absorbers for modern powertrains. Sedaghati et al. (2015,[18]) developed a torsional vibration damper by incorporating conventional centrifugal pendulum absorber and magnetorheological damper. Mayet et al. (2015,[20]) derived the optimal linear and nonlinear detuning for tautochronic centrifugal

ISSN 2354-0575

pendulum vibration absorbers using average Hamiltonian formulation.

This paper presents a combined numerical and design of experiments based on Taguchi method for determining the optimal parameters of CPVAs. To this end, the torsional shaft CPVA systems are modeled.

Subsequently, the system equations of motion are derived using FEM. The system equations of motion are then solved using Runge-Kutta method to determine torsional vibration of output shaft with different input torsional moments. Numerical calculations are performed using MapleSIM 2016.1a. Finally, using Taguchi method, the optimal parameters of the CPVA are derived considering the vibration duration and stability criterion.

2. Shaft-CPVAs systems modeling

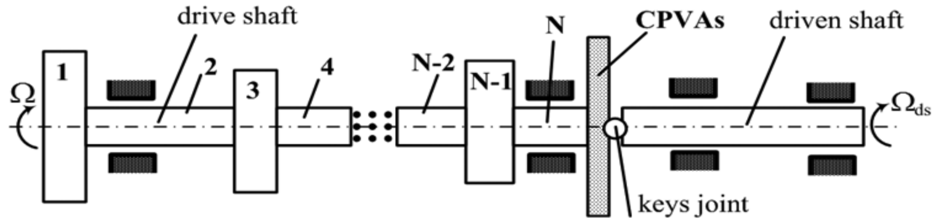

Figure 1. *Model of shaft-CPVAs system*

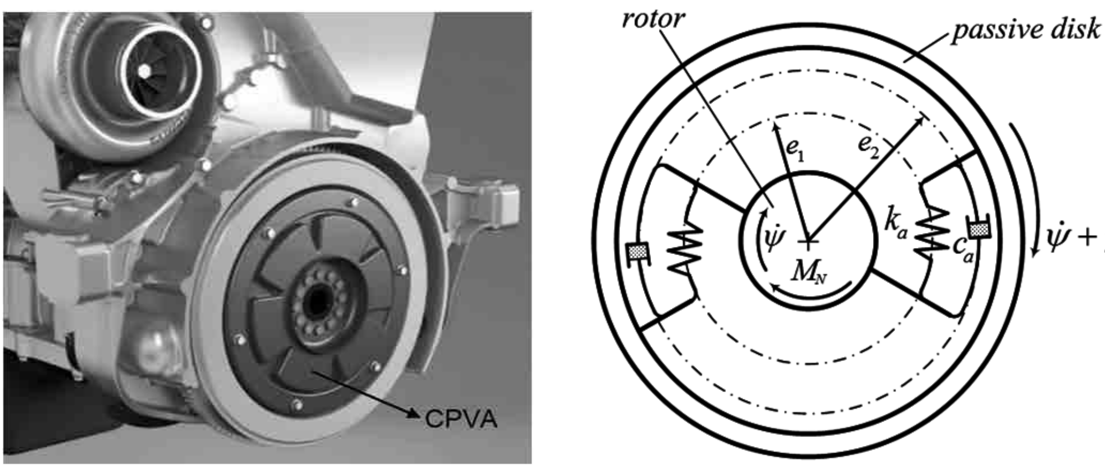

Figure 2. *Model of the CPVA with many absorbers*

The shaft-CPVAs system is modeled as shown in Figure 1. The time-varying torsional moment M^* is applied on the drive shaft, which is connected to the driven shaft through a CPVA. The stiffness and mass moment of inertia of drive shaft are J_s and k_s, respectively. The angular velocity of input drive shaft is Ω. Figure 2 shows the CPVAs, which contains a rotor connected to a passive disk via springs and dampers. Radius and mass moment of inertia of the rotor and passive disk are r, J_r, R, J_a, respectively. The stiffness of each spring is k_a.

The viscous coefficient of each damper is c_a. n_a represents the number of absorbers (an absorber consists of a spring and a damper). e_1 and e_2 indicate the radial position of spring and damper, respectively.

The relative rotation angle between the rotor and the passive disk is γ.

3. Equations of motion by FEM

Moment of the drive shaft is expressed as

$$M^* = 9{,}55\,.\,10^3\frac{P}{n},\text{Nm} \qquad (1)$$

where P and n are the power and rotational speed of the drive shaft.

Elements (1), (2), … (N) are shown in the figure 1.

The elemental equation for element (i) is

$$\begin{bmatrix} J_{si} & 0 \\ 0 & J_{si} \end{bmatrix}\begin{Bmatrix} \ddot{\varphi}_i \\ \ddot{\varphi}_{i+1} \end{Bmatrix} + k_{si}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{Bmatrix} \varphi_i \\ \varphi_{i+1} \end{Bmatrix} = \begin{Bmatrix} -M_i \\ M_{i+1} \end{Bmatrix} \qquad (2)$$

$i = 1..N-1$ and $M_1 = M^*$

The elemental equation for element (CPVAs) is (3)

$$\begin{bmatrix} J_r + J_a & J_a \\ J_a & J_a \end{bmatrix}\begin{Bmatrix} \ddot{\psi} \\ \ddot{\gamma} \end{Bmatrix} + n_a c_a e_2^2 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \dot{\psi} \\ \dot{\gamma} \end{Bmatrix} +$$

$$+ n_a k_a e_1^2 \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}\begin{Bmatrix} \psi \\ \gamma \end{Bmatrix} = \begin{Bmatrix} -M_N \\ 0 \end{Bmatrix}$$

On assembling equations (2) and (3) we get

$$\mathbf{M(q)\ddot{q} + C(q,\dot{q})\ddot{q} + K(q)q = F} \tag{4}$$

in where

$$\mathbf{M(q)} = \begin{bmatrix} J_{s1} & 0 & 0 & 0 & 0 & 0 \\ 0 & J_{s2} & 0 & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & J_{sN} & 0 & 0 \\ 0 & 0 & 0 & 0 & J_r + J_a & J_a \\ 0 & 0 & 0 & 0 & J_a & J_a \end{bmatrix}_{(N+2)} \tag{5}$$

$$\mathbf{C(q,\dot{q})} = n_a c_a e_2^2 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}_{(N+2)} \tag{6}$$

$$\mathbf{K(q)} = \begin{bmatrix} k_{s1} & -k_{s1} & \ldots & \ldots & \ldots & 0 \\ -k_{s1} & k_{s1} + k_{s2} & k_{s2} & \ldots & \ldots & 0 \\ \ldots & -k_{s2} & k_{s2} & \ldots & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\ 0 & 0 & 0 & 0 & 0 & n_a k_a e_1^2 \end{bmatrix}_{(N+2)} \tag{7}$$

$$\mathbf{F} = \left\{ -M_1 \quad 0 \quad \ldots \quad M_N \quad -M_N \quad 0 \right\}^T \tag{8}$$

$$\mathbf{q} = \left\{ \varphi_1 \quad \varphi_2 \quad \ldots \quad \varphi_N \quad \psi \quad \gamma \right\}^T \tag{9}$$

4. Determination of optimal parameters for the CPVAs

This section presents an optimization design of the CPVA's parameters using the Taguchi method. The Taguchi method proposes a special orthogonal arrays design to investigate the entire parameter space with a small number of experiments. The experimental results are then transformed into a signal-to-noise (S/N) ratio, which is used to measure the quality characteristics deviating from the desired values [9,22].

Objective functions of the optimization design are torsional vibration amplitude, vibration duration and shaft stability. Six design parameters will be investigated including the stiffness of spring (k_a), the viscous coefficient of damper (c_a), the mass moment of inertia of the passive disk (J_a), the number of absorbers (n_a) and radial position of spring (e_1) and damper (e_2). The design parameters are introduced in following form

$f_1 = \frac{e_1}{r}$ (10) $\quad f_2 = \frac{e_2}{r}$ (11) $\quad f_3 = \frac{J_a}{J_s}$ (12)

$f_4 = n_a$ (13) $\quad f_5 = \frac{k_a}{k_s}$ (14) $\quad f_6 = \frac{c_a}{k_s}$ (15)

In this paper, the number of design parameters is six and the level of each parameter is five. Therefore, 25 experiments are selected for the CPVA parameter design (L_{25} array table). For illustration of the proposed method, calculation will be performed for the input and output shaft parameters shown in Table 1 including *ith* diameter (d_i), *ith* length (L_i) and *ith* inertial mass moment (J_{si}).

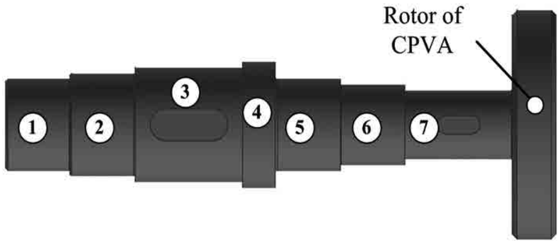

Figure 3. *The shaft-CPVA in this study*

Table 1. *Shaft parameters*

Parameters	d_i (mm)	L_i (mm)	J_{si} (kgmm^2)
Element (1)	40	30	59.188
Element (2)	45	30	94.807
Element (3)	50	50	240.835
Element (4)	55	20	141.043
Element (5)	40	30	59.188
Element (6)	35	30	34.695
Element (7)	30	50	31.212
Element rotor	100	20	1541.344
Shaft+Rotor	-	-	109459.46

The drive shaft rotates at a constant speed of 3000 r/min. The harmonic torsional moment applied on passive disk is $M(t) = 10^{-2} \sin \Omega t$. The maximum value of parameters e_1 and e_2 is r. Thus, f_1 and f_2 are selected from 0.2 to 1.0 with an incremental value of 0.2. The ratio between the mass moment of inertia of the passive disk and shaft (f_3) is selected from 0.02 to 0.1. The parameter f_4, which is equal to the number of absorbers (n_a) is chosen from 2 to 10. Regarding the parameter f_5, the stiffness of spring that connects the rotor and the passive disk is determined by

$$k_a = \frac{G_a d^4}{8D^3 s} \tag{16}$$

where D and d are mean diameter and wire diameter of the spring. s indicates number of coils of the spring. The spring parameters are selected as D = 4 mm, s = 10 and L = 30 mm. The wire diameter d is selected from 0.2 to 1.0 mm with incremental value of 0.2 mm. Shear modulus of the spring is G_a=7.85 104 MPa. Table 3 shows the calculated spring stiffness ka and dimensionless stiffness f_5.

These damping oils are often used in

ISSN 2354-0575

vibration absorber included SAE 0W-30, SAE 5W-40, SAE 10W-40, SAE 15W-40 and SAE 10W-60 [25]. Due to the relative velocity between the passive disk and rotor which is not great so that damping oil temperature working range of about 40°C [25] so we have data as table 2.

Table 2. *Vicosity oil dampers*

Type	Viscosity [Pa.s]	f_6
SAE 0W-30	$55.926.10^{-3}$	1.377E-06
SAE 5W-40	$76.551.10^{-3}$	1.885E-06
SAE 10W-40	$79.330.10^{-3}$	1.953E-06
SAE 15W-40	$91.057.10^{-3}$	2.242E-06
SAE 10W-60	$135.52.10^{-3}$	3.337E-06

From the level of six parameters in 25 experiments (Table 4), Taguchi proposed an optimal combination for the set of experimental parameters as shown in Table 5.

Table 3. *Spring stiffness and parameter*

Parameters	Values				
d (mm)	0.2	0.4	0.6	0.8	1.0
k_a (N/m)	64.68	1035	5239	16560	40429
f_5	0.987	0.404	0.128	0.025	0.001

Table 4. *Parameter range and calculated parameter levels*

Parameters	Range	Level				
		1	2	3	4	5
f_1	0.2 to 1	0.2	0.4	0.6	0.8	1.0
f_2	0.2 to 1	0.2	0.4	0.6	0.8	1.0
f_3	0.02 to 0.1	0.02	0.04	0.06	0.08	0.1
f_4	2 to 10	2	4	6	8	10
f_5	0.001 to 1.4	0.001	0.025	0.128	0.404	0.987
f_6	3.337E-06	1.377E-06	1.885E-06	1.953E-06	2.242E-06	3.337E-06

Table 5. *Experimental layout using an L_{25} orthogonal array proposed by Taguchi*

No. of experiments	Level of parameters					
	f_1	f_2	f_3	f_4	f_5	f_6
1	1	1	1	1	1	1
2	1	2	2	2	2	2
…	…	…	…	…	…	…
24	5	4	3	2	1	5
25	5	5	4	3	2	1

Using the parameter levels and combination scheme in Tables 4 and 5, the data of 25 input parameters are obtained as shown in Table 6. Then, torsional vibration of the system is determined by solving the system equation of motions.

Table 6. *L_{25} array design*

No. of experiments	Value of parameters					
	f_1	f_2	f_3	f_4	f_5	f_6
1	0.2	0.2	0.02	2	0.001	1.377E-06
2	0.2	0.4	0.04	4	0.025	1.885E-06
…	…	…	…	…	…	…
24	1.0	0.8	0.06	4	0.001	3.337E-06
25	1.0	1.0	0.08	6	0.025	1.377E-06

In order to determine the optimal parameters and influence of each parameter on the CPVA performance, the S/N ratio will be determined using commercial statistical software package MINITAB 17. In the Taguchi method, the S/N ratio represents the desirable signal value and the undesirable noise value, which is used to transform the quality characteristics [22]. The equation of S/N ratio depends on the criteria used for optimization process. In this study, the S/N ratio for the *smaller-is-better* is used and calculated by

$$S/N = 10\log E \qquad (17)$$

where E is the mean of sum of squares of observed data calculated by

$$E = \frac{1}{N} \sum y^2 \quad (18)$$

where N is the number of measurements in an experiment, in this case, N = 3 and y is the respective simulated vibration responses. The optimal level of design parameters is the level with the greatest S/N value. Figure 4 shows the S/N ratio and corresponding relative rotation angle between the drive and driven shaft calculated at different times t_1 = 0.1 s; t_2 = 0.15 s and t_3 = 0.2 s. Tables 7 show the average signal-to-noise ratios determined for each level.

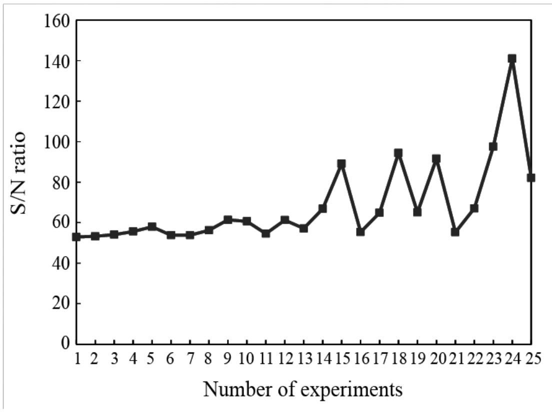

Figure 4. *Selection criteria for calculating S/N ratio*

According to the Taguchi method, the statistic delta defined as the difference between the maximum and minimum mean response was used to determine the most influencing factor. The rank in Table 7 indicates the rank of each delta, where the first rank corresponds to the largest delta (delta is the difference between the maximum and minimum mean response across levels of a factor).

Table 7. *Influence levels of the design parameters*

Level	f_1	f_2	f_3	f_4	f_5	f_6
1	54.7	54.4	68.4	72.9	80.8	62.2
2	57.2	60.0	71.7	73.4	69.1	64.6
3	65.8	71.9	79.0	64.3	59.2	65.4
4	74.2	77.9	62.1	69.6	65.0	66.6
5	88.6	75.3	59.3	60.1	66.3	81.7
Delta	33.8	23.5	19.7	13.3	21.5	19.4
Rank	1	2	4	6	3	5

5. Results and Discussion

5.1. Optimal parameters of CPVA

From figure 4, the set of parameters #24 gives greatest value of S/N ratio. Thus, it will be selected for simulation of system torsional vibration responses. Table 7 illustrates that shows the highest effect on the torsional vibration of the system, subsequently to f_2, f_5, f_3, f_6, and f_4.

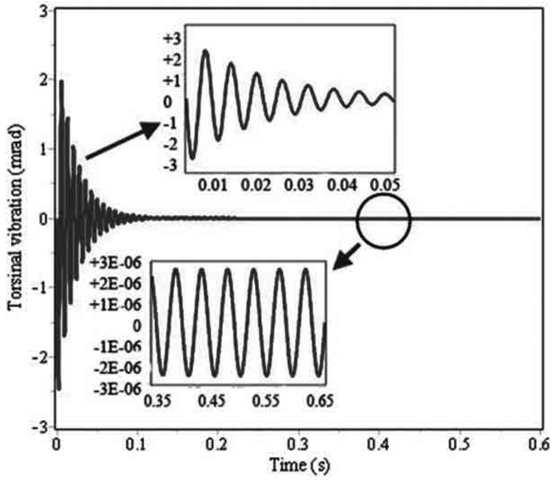

Figure 5. *Torsional vibration of shaft with the 24th set parameters, a) with CPVA and without CPVA, b) with CPVA only*

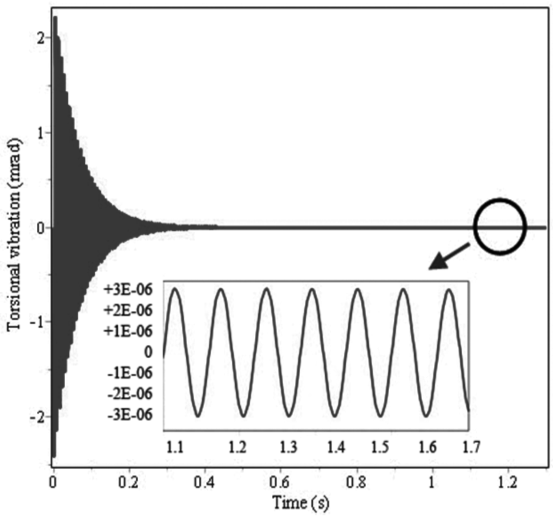

Figure 6. *Torsional vibration of shaft with the 18th set parameters*

To show the performance of the system with proposed disgn paramters, Figure 5 displays the torsional vibration responses of the shaft assembled with CPVA using the set parameters #24. For the purpose of comparison, Figures 6 and 7 shows the vibration responses using set parameters #18 and 13, respectively.

Figures 5-7 show that the time for vibration cancelation are approximate 0.1 s, 0.4 s and 2.5 s for set parameters #24, #18 and 13, respectively. Thus the effectiveness of set parameters #24 is highest among the others. This is consistent with the results depicted in figure 4.

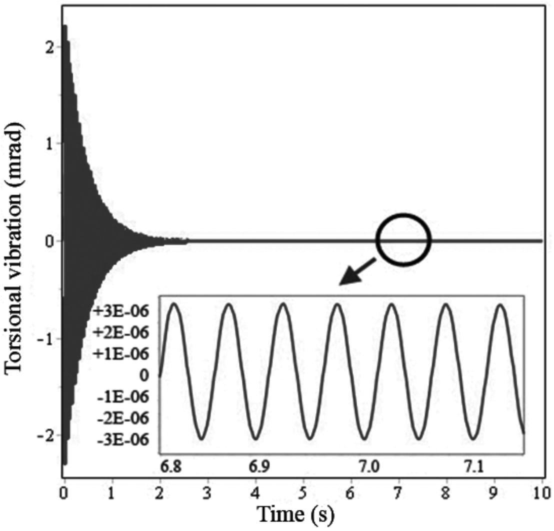

Figure 7. *Torsional vibration of shaft with the 13th set parameters*

5.2. Effect of rotational speed on torsional vibration of the shaft

Use the CPVAs with the optimal parameters (24th) and change the speed of rotation of the shaft in the region from 500 to 3000 r/min, the results obtained was shown as Table 8 and the graph from Figure 8. It is seen that the higher rotational speeds, the higher torsional vibration amplitudes.

Table 8. *Torsional vibration of the shaft as function of speed*

n (r/min)	$\theta(t)\|_{t=0.1s}$(rad)	$\theta(t)\|_{t=0.2s}$(rad)
500	5.981E-07	-2.382E-08
1000	1.198E-06	-4.552E-08
1500	1.799E-06	-6.722E-08
2000	2.399E-06	-8.891E-08
2500	2.999E-06	-1.106E-07
3000	3.599E-06	-1.323E-07

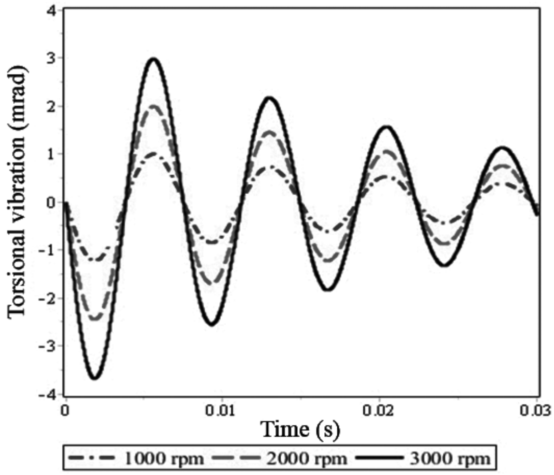

Figure 8. *Torsional vibration of shaft with optimal parameters at different speeds*

5. Conclusions

This article presents a combined methods Runge-Kutta algorithm and Design of Experiments (DOE) with Taguchi method for determining the optimal parameters toreducetorsional vibration for shaft that used CPVA. The objective functions are cancellation time of vibration and rotation stability. The calculationsare completed on MapleSIM 2016.1a and Minitab 17 software that made reliable results. In further research, the authors use analytical methods to specify optimal parameters of the CPVA.

Acknowledgement

This research was financially supported by Hung Yen University of Technology and Education.

References

[1]. C.-T.Lee, S.W.Shaw, *A Subharmonic Vibration Absorber for Rotating Machinery*, ASME Journal of Vibration and Acoustics, Vol.119, pp.590-595, 1997.
[2]. C.-P.Chao, S.W.Shaw, *The Efects of Imperfections on the Performance of the Subharmonic Vibration Absorber System*, Journal of Sound and Vibration, Vol.215, pp.1065-1099, 1998.
[3]. C.-P. Chao, S.W.Shaw, C.T.Lee, *Stability of the Unison Response for a Rotating System with Multiple Centrifugal Pendulum Vibration Absorbers*, ASME Journal of Applied Mechanics, Vol. 64, pp.149-156, 1997.
[4]. C.-P.Chao, C.T.Lee, S.W.Shaw, *Non-unison Dynamics of Multiple Centrifugal Pendulum Vibration Absorbers*, Journal of Sound and Vibration,Vol.204, pp.769-794, 1997.
[5]. C.-P.Chao, S.W.Shaw, *The Dynamic Response of Multiple Pairs of Subharmonic Torsional Vibration Absorbers*, Journal of Sound and Vibrations Vol. 231(2), pp.411-431, 2000.
[6]. A.-S.Alsuwaiyan, S.W.Shaw, *Performce and Dynamic Stability of General-path Centrifugal Pendulum Vibration Absorbers*, Journal of Sound and Vibrations, Vol. 252(5), pp.791-815, 2002.
[7]. C.-T.Lee, S.W.Shaw, V.T.Coppola, *A subharmonic Vibration Absorber for Rotating Machinery*,

ASME Journal of Applied Mechanics, Vol.119, pp.590-595,1997.
[8]. K.D.Dien, V.X.Truong, N.D.Chinh, *Research to Reduce Vibration for Shaft of Machine using the Reduced Vibration CPVA*, Proceedings of the Regional Conference on Mechanical and Manufacturing Engineering, Part B, pp.132-136, 2014.
[9]. W.-H. Yang,Y.S.Tarng, *Design Optimization of Cutting Parameters for turning operations based on the Taguchi method*, Journal of Materials Processing Technology, Vol.84, pp.122-129, 1998.
[10]. J.-A.Gani, I.A.Choudhury, H.H.Hassan, *Application of Taguchi Method in the Optimization of End Milling Parameters*, Journal of Materials Processing Technology, Vol.145, pp.84-92, 2004.
[11]. A.-S.Alsuwaiyan, S.W.Shaw, *Performance and Dynamic Stability of General-path Centrifugal Pendulum Vibration Absorbers*, Journal of Sound and Vibration, Vol.252, pp.791-815, 2002.
[12]. B.-C. Carter, *Rotating Pendulum Absorbers with Partly Solid and Liquid Inertia Members with Mechanical or Fluid Damping*, British Patent, No.337, pp.466, 1929.
[13]. E.-T.Taylor, *Eliminating Crankshaft Torsional Vibration in Radial Aircraft Engines*, SAE Technical, Paper 360105, 1936.
[14]. C.-T.Lee; S.W.Shaw, *Comparative Study of Nonlinear Centrifugal Pendulum Vibration Absorbers*, ASME Applied Mechanics Division, Publiccations-AMD, Vol.192,pp.91-91,1994.
[15]. J.-F.Madden, F.John, *Constant Frequency Bifilar Vibration Absorber,* US Patent, No. 4, Vol. 218, 1980.
[16]. S.-W.Shaw, P.M.Schmitz, A.G.Haddow, *Tautochronic Vibration Absorbers for Rotating Systems*, ASME Journal of Computer Nonlinear Dynamics, Vol.1(1), pp.283-293,2006.
[17]. M.-Swank, P.Lindemann, *Dynamic Absorbers for Modern Powertrains*, SAE Technical, Paper.2011-01-1554, 2011.
[18]. E.-H.Abouobaia, R.Bhat, R.Sedaghati, *Development of A New Torsional Vibration Damper Incorporating Conventional Centrifugal Pendulum Absorber and Magnetorheological Damper,* Journal of Intelligent Material Systems and Structures, Special Issue, pp.1-13,2015.
[19]. J.-P.Den Hartog, *Mechanical Vibrations*, Dover Publications, 1985.
[20]. J.Mayet, D.Rixen, H.Ulbrich, *Experimental Investigation of Centrifugal Pendulum Vibration Absorbers*, 11th International Conference on Vibration Problems, Z. Dimitrovova ´ et.al. (eds.), Lisbon, Portugal, 9–12 September 2013
[21]. http://www.viscopedia.com/viscosity-tables/substances/engine-oil/
[22]. Lead Technologies Inc, Minitab 17 Helps.
[23]. Waterloo Inc, MapleSIM 2016.1a Helps.

MỘT CÁCH TIẾP CẬN MỚI TRONG TỐI ƯU HÓA THIẾT KẾ BỘ HẤP THỤ DAO ĐỘNG CPVAs GIẢM DAO ĐỘNG XOẮN CHO TRỤC

Tóm tắt:
Bài báo trình bày kết hợp phương pháp giải tích và thiết kế thực nghiệm để xác định các tham số tối ưu của bộ hấp thụ dao động CPVAs chẳng hạn độ cứng lò xo, cản nhớt của dầu giảm chấn, mômen quán tính của hệ thống, số bộ giảm chấn và vị trí lắp đặt. Trước tiên hệ phương trình vi phân dao động của cơ hệ được thiết lập và phân tích dựa trên phương pháp phần tử hữu hạn và thuật toán RKA. Sau đó áp dụng phương pháp Taguchi để xác định các tham số tối ưu của bộ CPVAs, từ đó tối ưu hóa được thiết kế của bộ CPVAs. Kết quả nghiên cứu được áp dụng giảm dao động xoắn cho trục máy có kết cấu bất kỳ nhằm giảm dao động có hại cho trục, nâng cao độ bền và khả năng làm việc cho trục.
Từ khóa: *Bộ hấp thụ dao động CPVAs, dao động xoắn, thông số tối ưu, FEM, Runge-Kutta, phương pháp tối ưu Taguchi.*